NOTE ON A PROBLEM OF FRÉCHET*

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1. *Introduction.* Fréchet has proposed the following problem: Characterize the most general space in which there exists a non-constant real-valued continuous function. Its solution has been given by Urysohn† for the spaces of Hausdorff and by Chittenden‡ for topological spaces.

However, in his generalization, Chittenden used for his definition of a continuous function a neighborhood definition§ of Fréchet which is not entirely adequate for the general topological space. That the definition is not adequate is easily shown by the following example. Let the space (P, K) be a set of points corresponding to the open interval $[0, 1]$. For any set E of (P, K), $K(E)=R(E)+J(E)$. A point x is in $R(E)$ if x is in a derived set of E under the metric relationship. A point x is in $J(E)$ if E contains both x and the open interval $[0, x/2]$. A neighborhood of x is any set to which x is interior. The correspondence $f(x)=x$ between the space (P, K) and the space (P, R) determines under the Fréchet definition a biunivocal bicontinuous transformation, although the two spaces are not homeomorphic.

The purpose of this paper is to solve the problem for the general topological space using the Sierpinski definition|| of a continuous transformation. The notation used will be that of Chittenden.¶

2. *Thoroughly Interior.* A point a is said to be *thoroughly in-*

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* Presented to the Society, July 18, 1933.

† Urysohn, *Über de Mächtigkeit der zusammenhängenden Mengen*, Mathematische Annalen, vol. 94 (1925), p. 290.

‡ Chittenden, *On general topology and the relation of the properties of the class of all continuous functions to the properties of space*, Transactions of this Society, vol. 31 (1929), p. 310.

§ Chittenden, loc. cit., p. 309.

|| Sierpinski, *La notion de dérivée comme base d'une théorie des ensembles abstraits*, Mathematische Annalen, vol. 97 (1926), pp. 321–337.

¶ Chittenden, loc. cit. Also see Stephens, *Continuous transformations of abstract spaces*, Transactions of this Society, vol. 34 (1932), p. 395.