BANACH ON LINEAR OPERATIONS

Théorie des Opérations Linéaires. By Stefan Banach. Warsaw, Monografje Matematyczne, Vol. I, 1932. vii+254 pp.

The present book, the first of the series published by the Monografje Matematyzcne, is an enlarged and considerably modified translation of the *Teorja Operacyi* (in Polish) by Banach. It represents a noteworthy climax of long series of researches started by Volterra, Fredholm, Hilbert, Hadamard, Fréchet, F. Riesz, and successfully continued by Steinhaus, Banach, and their pupils. In a short review it is impossible to give even an approximate idea of the richness and importance of the material, entirely new to a large extent, which is gathered in this book. Roughly speaking the book treats of the field which was touched upon by Hildebrandt in his excellent address on linear functional transformations in general spaces,* but naturally is considerably more extensive and intensive. The theory of linear operations is a fascinating field in itself but its importance is still more emphasized by numerous beautiful applications to various problems of analysis and function theory (real and complex). A number of these applications are contained in the book; a still larger number of them, increasing from year to year, are being inspired by the ideas developed in the book. An attentive reader will find a constant source of inspiration in this monograph which undoubtedly will exercise a great influence on the further development of the subject. A description of the contents follows.

In the Introduction (19 pp.) the author recalls some basic properties of Lebesgue and Stieltjes integrals, and of metric spaces. Particular attention is given to subsets of such spaces, which either are Borel measurable or possess the property of Baire, and also to Borel measurable operations transforming one metric space into another. In Chapter 1 (*Groupes*, 6 pp.) the author specializes the metric space under consideration by adding the usual postulates for groups (with respect to the operation "+", with inverse designated by "−") and also the two postulates concerning limits:

$$x_n \to x \quad \text{implies} \quad (-x_n) \to (-x);$$

$$x_n \to x \text{ and } y_n \to y \quad \text{implies} \quad (x_n + y_n) \to (x + y).$$

It is proved that every subgroup which possesses Baire's property and is of the second category coincides necessarily with the whole space (assumed to be complete). The additive and linear (additive+continuous) operations are introduced at this early stage. It is proved that the set where a sequence $\{U_n(x)\}$ of linear operations converges, or even where it is only bounded, is either the whole space or a set of the first category (the space is assumed to be connected). Chapter 2 (*Espaces vectoriels généraux*, 9 pp.) deals with linear spaces and additive operations defined on such spaces. The central point here is an important theorem concerning a possibility of extending to the whole space

* This Bulletin, vol. 37 (1931), pp. 185–212.