ON THE ELEMENT OF DECOMPOSITION OF A DOUBLY PERIODIC FUNCTION OF THE SECOND KIND*

BY M. A. BASOCO

1. *Introduction.* Hermite† has shown that a meromorphic function which satisfies periodicity relations of the form

$$(1) \qquad F(z + 2\omega) = \mu F(z), \qquad F(z + 2\omega') = \mu' F(z),$$

where $\omega'/\omega = a + ib$, $b > 0$, and μ, μ' are independent of z, may be expressed in terms of the function

$$(2) \qquad G(z) = \frac{\sigma(z+\lambda)}{\sigma(z)\sigma(\lambda)} e^{\rho z},$$

and its derivatives, in which λ, ρ are suitably determined constants and $\sigma(u)$ is the Weierstrass sigma function. The class of functions which satisfy conditions (1) has been called by Hermite doubly periodic of the second kind. We shall exclude from the present considerations Mittag-Leffler's singular case‡ for this category of functions. He has shown that if $F(z) = f(z)e^{\rho z}$, where $f(z)$ is an elliptic function, then the suitable element of decomposition is $e^{\rho z}\zeta(z)$, where $\zeta(z)$ is the zeta function of Weierstrass.

In what follows we shall be concerned with the nature of the power series development§ of (2) which we shall henceforth write in the form

$$(3) \qquad \Phi(u, v) = \frac{\sigma(u+v)}{\sigma(u)\sigma(v)} e^{\rho z},$$

* Presented to the Society, June 23, 1933.

† Hermite, *Sur quelques applications des fonctions elliptiques*, Oeuvres, vol. 3, Chaps. 1, 2, pp. 266 et seq. See also the monograph of Appell, *Sur la décomposition d'une fonction méromorphe en éléments simples*, Mémorial des Sciences Mathématiques, Fascicule 36, 1929.

‡ Comptes Rendus, vol. 90 (1880), p. 178.

§ See Krause, *Doppeltperiodischen Funktionen*, vol. 1, pp. 299–308. Halphen, *Fonctions Elliptiques*, vol. 1, Chap. 7.