ON THE USE OF CESÀRO MEANS IN DETERMINING CRITERIA FOR FOURIER CONSTANTS*

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1. *Introduction.* In the portion of Professor Fejér's address dealing with Fourier series, the positiveness of certain linear operations played a central role. These linear operations were those that arise in forming for a series the partial Cesàro means of various integral orders.

The positiveness of the linear functional operation in the case of a wide class of Fourier series resulted from the positiveness in the case of certain very simple trigonometric series. One such series is the series

$$\frac{1}{2} + \cos\theta + \cos 2\theta + \cos 3\theta + \cdots + \cos n\theta + \cdots, \tag{1}$$

whose behavior is of fundamental importance in studying the convergence or summability of the general Fourier series. For the series (1) the partial Cesàro mean of the first order takes the form

$$\frac{1}{2n}\cdot\frac{\sin^2(n\theta/2)}{\sin^2(\theta/2)}, \qquad (\theta \neq 2m\pi), \tag{2}$$

and the value $n/2$ for the excepted values of θ, thus being obviously non-negative for all values of θ.

For many questions involving Fourier series further light is shed on the behavior of the series by considering the Cesàro means of non-integral order introduced by Knopp, M. Riesz, and Chapman. These means may be defined as follows. We set

$$A_0^{(k)} = 1,\ A_n^{(k)} = \frac{(k+1)\cdots(k+n)}{n!}, \qquad (n = 1, 2, \cdots), \tag{3}$$

so that

$$(1-z)^{-(k+1)} = \sum_{n=0}^{\infty} A_n^{(k)} z^n, \qquad (|z| < 1),$$

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