CERTAIN PROBLEMS OF CLOSEST APPROXIMATION*

BY DUNHAM JACKSON

1. *Introduction.* In connection with the theory of systems of polynomials which are orthogonal and normalized with respect to a given weight function an important question is that of the order of magnitude of the nth polynomial of the sequence as n becomes infinite. In the fundamentally important case of Jacobi polynomials† as well as for the systems of polynomials corresponding to much more general weight functions‡ asymptotic formulas show that the polynomials of the normalized sequence remain bounded, at least in the interior of the interval for which they are constructed. This paper is in part devoted to a much less profound but considerably broader study of upper bounds for the order of magnitude of the polynomials under still more general hypotheses as to the character of the weight function. It is believed to be of interest by reason of the simplicity of the methods employed, and their ready applicability to the obtaining of results with regard to the behavior of the polynomials even at points where the weight function vanishes.

Attention is given also to similar problems in the case of trigonometric sums, which are in some respects more readily accessible to treatment than polynomials.

The rest of the paper is concerned with the convergence of the development of a given function in series of the orthogonal polynomials or trigonometric sums, or more directly, as the terms of the series do not enter explicitly into the calculation, with

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† See G. Darboux, *Mémoire sur l'approximation des fonctions de très grands nombres, et sur une classe étendue de développements en série*, Journal de Mathématiques, (3), vol. 4 (1878), pp. 5–56.

‡ See G. Szegö, *Über den asymptotischen Ausdruck von Polynomen, die durch eine Orthogonalitätseigenschaft definiert sind*, Mathematische Annalen, vol. 86 (1922), pp. 114–139; S. Bernstein, *Sur les polynomes orthogonaux relatifs à un segment fini*, Journal de Mathématiques, (9), vol. 9 (1930), pp. 127–177, and vol. 10 (1931), pp. 219–286.