ON THE SUMMABILITY AND GENERALIZED SUM OF A SERIES OF LEGENDRE POLYNOMIALS*

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1. *Introduction.* The results obtained in this paper are as follows.

(A) *The series of Legendre polynomials* $\sum n^p X_n(x)$, *where* p *is a positive integer, is summable* (H, p) *for* $-1<x<1$, *and summable* $(H, p+1)$ *for* $-1 \leqq x<1$.

(B) *The generalized sum over the range* $-1<x<1$ *is*

$$\sum_1^\infty n^p X_n(x) = -\frac{1}{2(2y)^{p-1/2}} \begin{vmatrix} 2 & 0 & 0 & 0 & \cdots 0 & 1 \\ y & 2y & 0 & 0 & \cdots 0 & 1 \\ A_2^3 & y & 2y & 0 & \cdots 0 & 1 \\ A_3^4 & A_2^4 & y & 2y & \cdots 0 & 1 \\ \cdot & \cdot & & & \cdot & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ A_{p-2}^{p-1} & A_{p-3}^{p-1} & \cdots & y & 2y & 1 \\ A_{p-1}^{p} & A_{p-2}^{p} & \cdots & A_2^p & y & 1 \end{vmatrix}$$

where

$$y = 1 - x; \qquad A_t^p = {}_pC_t + (-1)^t {}_{p-1}C_t; \qquad (p > 2).$$

2. *The Cases* $p=0, 1, 2$. We first obtain these results for $p=0$, 1, 2. Let p be a positive integer, $S_{n,p}$ the sum of the first n terms of the series $\sum n^p X_n(x)$, $S_{n,p}^{(p)}$ the pth Hölder mean of $S_{n,p}$, and $S^{(p)}$ the limit of this mean for $n \to \infty$.

The generating function of the Legendre polynomials gives us at once the sum of the convergent series

$$(1) \qquad \sum_1^\infty X_n(x) = S^{(0)} = [2(1-x)]^{-1/2} - 1, \qquad (-1 < x < 1).$$

We can readily find $S^{(1)}$ by use of the recursion formula

$$(2) \qquad (2m+1)xX_m = (m+1)X_{m+1} + mX_{m-1},$$

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