alizes the theorem of Thompson and Tait. We can prove, in fact, that a condition for an affirmative answer to our question is that, on any tube of (S), either all or none of the transversal curves should be closed.

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ON THE CONDITION THAT TWO ZEHFUSS MATRICES BE EQUAL

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1. *Introduction.* In a recent paper* Williamson has considered matrices whose sth compounds are equal. The present paper considers the somewhat analogous problem of finding the conditions that two Zehfuss matrices be equal.

Suppose that R is a matrix of n_1 rows and m_1 columns whose ijth element is r_{ij}, and that P is another matrix of n_2 rows and m_2 columns. Now, if the matrix Q of n_1n_2 rows and m_1m_2 columns can be partitioned into submatrices each of n_2 rows and m_2 columns such that the ijth submatrix is $r_{ij}P$, then Q is a *Zehfuss matrix*† or the *direct product matrix*‡ of R and P. We shall write

$$Q = R\langle P\rangle = \langle P\rangle R.$$

In general, however, $R\langle P\rangle \neq \langle P\rangle R$.

It is the purpose of this paper to find out under what conditions the matrix equation

$$A\langle B\rangle = C\langle D\rangle$$

is true. That is, we shall find the most general form of the matrices A, B, C, D when the above equation holds.

2. *The Simplest Case.* We shall begin by considering the simplest case, where A, B, C, D are row vectors, where A and D are of order m_1, where B and C are of order m_2, and where

$$(m_1, m_2) = 1;$$

that is to say, m_1 and m_2 are prime to one another. Suppose that

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* J. Williamson, this Bulletin, vol. 39 (1933), p. 109.

† G. Zehfuss, Zeitschrift für Mathematik und Physik, vol. 3 (1858), p. 298.

‡ L. E. Dickson, *Algebras and Their Arithmetics*, p. 119.