ON SECTION A OF PRINCIPIA MATHEMATICA†

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1. *Introduction*. Section A of Whitehead and Russell's *Principia Mathematica*, which is concerned with the "theory of deduction" for "elementary" propositions, consists of "formal" (or "official") propositions and of "informal" (or "unofficial") statements. In previous papers,‡ I discussed the theory of deduction when that theory is considered as given solely by the "formal" propositions. Huntington§ has recently given sets of postulates for "the algebra of logic" (or "Boolean algebra," or "the classic logic of classes," or "the logic of classes");‖ and he obtained, in connection with these postulate-sets, two "informal" systems, consisting of "formal" and of "informal" propositions of Section A. I propose to bring out certain facts concerning Section A in view of Huntington's findings. My main results are that the "formal" Section A, as given by Huntington or by me, is derivable from the logic of classes; that this "formal" Section A is not the *whole* of the logic of classes; and that Huntington's "informal" systems are not adequate either for the classic logic of classes or for "the classic logic of propositions."¶

2. *The "Formal" Section* A *Derivable from the Logic of Classes.* Huntington's sixth set of postulates for Boolean algebra is expressed in terms of the following undefined ideas: a *class* K, a *subclass* T of K, a *binary operation* $+$, and a *unary operation* $'$. In this set, Postulates 6.1, 6.2, 6.3, 6.4, 6.5, 6.6, 6.7 correspond

† Presented to the Society, March 18, 1933.

‡ See especially this Bulletin, vol. 37 (1931), pp. 480–488, and vol. 38 (1932), pp. 589–593. I shall refer to these papers later as Paper 1 and Paper 2, respectively.

§ See the Transactions of this Society, vol. 35 (1933), pp. 274–304. Later references to Huntington will imply this paper.

‖ I use these terms interchangeably for the general logic of classes as developed by Boole, modified by Schröder, and formulated postulationally by Huntington in his *Sets of postulates for "the algebra of logic."*

¶ I use the term "the classic logic of propositions," or simply "the logic of propositions," for Schröder's *Aussagenkalkül* as formulated in my *Sets of postulates for the logic of propositions*, Transactions of this Society, vol. 28 (1926), pp. 472–478.