A COMPLETE CENSUS OF 4×4 MAGIC SQUARES*

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A complete enumeration of magic squares of order 3 is easy to make and the result turns out to be 72. In this enumeration two squares are considered different if they do not have the same numbers in corresponding cells. In this and in the study of 4×4 squares we define a magic square as one in which the elements in every row and column add up to the same sum, this sum being given for the square of order n by the formula

$$\frac{n(n^2+1)}{2}.$$

With this definition a magic square remains magic after any permutation of the rows among themselves, or of the columns among themselves, or by an interchange of rows and columns. This associates with any magic square a set of $2(n!)^2$ squares all of which are magic.

By a permutation of the rows and columns of any square the largest entry in the square may be transferred to any particular cell; in particular it may be placed in the lower left hand corner. The other rows and columns may then be permuted so that the entries in the bottom line and in the left hand column read in descending order of magnitude. Also by an interchange of rows and columns, if necessary, the element in the bottom row next to the left hand corner may be made larger than the element in the left hand column which is next above the left hand corner. No further adjustments are then possible and the square will be said to be *normalized*. Thus the square

22	5	8	11	19		3	21	19	12	10
4	7	15	18	21		6	4	22	20	13
6	14	17	25	3	becomes	14	7	5	23	16
13	16	24	2	10		17	15	8	1	24
20	23	1	9	12		25	18	11	9	2

after normalization;

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