ABSTRACT IDEAL THEORY*

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1. *Introduction.* Abstract ideal theory is a branch of algebra which has come into prominence only in recent years; its importance for algebraic problems and also for branches of mathematics outside of algebra proper has however been increasingly recognized; it seems established that the ideals, corresponding in many ways to the normal subgroups in the theory of groups, are the most convenient building stones in a large number of algebraic structures.

In the following I have tried to give a survey of the most important problems and results, but it should be realized that an account of this kind must necessarily be incomplete, since the field is too wide and too diverse to be covered in a single lecture. To limit the subject, only commutative ideal theory will be considered; demonstrations have been omitted, although reluctantly, since an occasional proof will often clarify more than any explanation the tools and working methods of a theory. Another difficulty in this case lies in the fact that the theory itself to a large extent is still in an evolutionary stage and has not reached the harmonious form it will probably assume later on. Only for domains in which the finite chain condition holds does it seem to have arrived at some degree of perfection. Historically several of the fundamental ideas can be traced to the work of Dedekind, Kronecker, and Lasker, but the main contributions to the theory have been made in the last ten years by E. Noether, Krull, van der Waerden, Henzelt, Grell, Stiemke, and others.

The first part of the paper contains some of the main properties of ideals, operations on ideals, quotient rings, and isomorphisms; then follows a discussion of prime and primary ideals and the consequences of the finite divisor chain condition. The four principal ideal representations by E. Noether are treated fairly completely, and to illustrate these abstract de-

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* Symposium lecture delivered at the meeting of the Society in New York, April 14, 1933.