HANCOCK ON ALGEBRAIC NUMBERS

Foundations of the Theory of Algebraic Numbers. By H. Hancock. Vol. I. *Introduction to the General Theory.* 27+602 pp. Vol. II. *The General Theory.* 26+654 pp. New York, Macmillan, 1931–32.

Hancock's new book is easily the most voluminous treatise in the literature on algebraic numbers and ideal theory. The publication of such a handbook in English is in itself quite remarkable since ideal theory, both in algebraic fields and in abstract rings, mainly has been a continental or more specifically a German branch of mathematics. Textbooks in other languages have been few. Reid's book containing an elementary treatment of quadratic fields seems to have been the only contributions on the subject in English.

Since the time of Gauss the theory of algebraic numbers has been considered as one of the spires of mathematics and Hancock approaches his subject in an appropriate spirit: "By virtue of the simplicity of its foundations and the rigor of its deductions, arithmetic stands alone in the beauty and harmony of its truths. A divine gift, it offers proof that the mind is a reality attested by the sciences on the one hand, and the philosophies on the other. The province of arithmetic in this high position between science and philosophy, is both to serve and to be served in the quest of higher truths."

In the introduction the author discusses the natural problem of the foundations of ideal theory and decides to follow mainly the ideas of Dedekind, while their relations to the theories of Kronecker and Hensel are emphasized. This seems to be the simplest and most practical way of building up the ideal theory in algebraic fields. The relation between the various theories of Kummer, Dedekind, Zolotareff, Kronecker, Hensel, Prüfer, and others are fairly well known at the present time and a parallel development of all theories would be superfluous. Dedekind's point of view is in many ways both the simplest and the most general and this opinion, it seems to me, is confirmed by the facility with which the theory can be extended to abstract rings. It is true, however, that Hensel's theory has received a new impetus through the recent investigations of Krull on rings with generalized absolute values (Bewertungsringe).

Hancock's book deals exclusively with the classical ideal theory in algebraic fields. In the first volume one finds the foundations, beginning with the principal properties of polynomials, resultants, discriminants, and other preliminary notions. Then the definitions and main properties of fields, algebraic integers, units, bases, discriminants, and applications to quadratic and cubic fields are given, all following the well established lines of the theory. In the Chapters 5–7 one obtains a detailed discussion of the algebraic moduli introduced by Dedekind and in a form which is almost identical to the presentation in the supplements of the fourth edition of Dirichlet's *Number Theory.* The following Chapter 8 contains the elements of Kronecker's modular systems. In my opinion it would have been wiser to omit these chapters on modular theory in volume I and include them at the beginning of volume II where they are applied for the first time. Volume I would then have contained only