CONCERNING COMPACT CONTINUA IN CERTAIN SPACES OF R. L. MOORE

BY J. H. ROBERTS

While writing his colloquium book, *Foundations of Point Set Theory*,† R. L. Moore noted that a large body of theorems concerning internal properties of compact continua could be established on the basis of a set of axioms (Axioms 1–5, *Foundations*) insufficient to make the space S itself a subset of a plane. He suggested that possibly every compact continuum M in S was homeomorphic with a compact continuum in the plane. In the present paper it is shown that, with possibly one exception, this is the case. If S is itself compact then it is homeomorphic with a subset of a sphere (possibly the sphere itself). But any compact continuum which is a *proper* subset of S is homeomorphic with a compact continuum in the plane.

THEOREM 1. *If S is a space in which Moore's Axioms* 1–5 *hold true and M is a closed and compact subset of S, then M is homeomorphic with a subset of a sphere. If furthermore M is a proper subset of S, then it is homeomorphic with a subset of a plane.*

Let E and J denote, respectively, a simple domain‡ and its boundary. It will be shown that if L is any circle in a plane and T_1 is any topological transformation of J into L, then there exists a topological transformation T_2 of $E \cdot M+J$ into a subset of L plus its interior, such that for each point P of J, $T_2(P) = T_1(P)$. From this result it readily follows that any closed and compact subset of S is homeomorphic with a subset of a sphere. If M is a closed and compact *proper* subset of S, and P is a point of $S-M$, then the closed and compact point set $M+P$ is homeomorphic with a subset of a sphere, and thus M is homeomorphic with a proper subset of a sphere, and hence homeomorphic with a subset of a plane. Henceforth it will be assumed that M is a closed and compact subset of $E+J$, where E is a simple domain and J is its boundary.

† Colloquium Publications of this Society, vol. XIII. Henceforth this book will be referred to as *Foundations*.

‡ A *simple domain* is a domain bounded by a simple closed curve.