ON IRREDUCIBLE SYSTEMS OF ALGEBRAIC DIFFERENTIAL EQUATIONS

BY E. GOURIN

Given two irreducible algebraic manifolds, if the first is a proper sub-manifold of the second, the first is of lower dimensionality than the second.* We prove here an analogous result for systems of algebraic differential equations.

Terminology and notation will be as in Ritt's monograph, *Differential Equations from the Algebraic Standpoint.*† Let Σ_1 be a non-trivial closed irreducible system. Let the unknowns be $u_1, \cdots, u_q, y_1, \cdots, y_p$, with the u_i arbitrary unknowns. We prove the following theorem.

THEOREM. *If the manifold of the non-trivial closed irreducible system Σ_1 is a proper sub-manifold of the manifold of another such system Σ_2, then either Σ_2 has a set of arbitrary unknowns of which $u_1, \cdots, u_q$ form a proper subset, or $u_1, \cdots, u_q$ form a complete set of arbitrary unknowns of the system Σ_2 and, for these arbitrary unknowns, resolvents of Σ_1 are of lower order than those of Σ_2.*‡

Because Σ_2 holds Σ_1, and Σ_1 is closed, every form of Σ_2 is contained in Σ_1. Then, certainly, Σ_2 cannot contain a form involving the u_i alone. Otherwise Σ_1 would contain such a form. Consequently, there exists in Σ_2 a set of arbitrary unknowns of which $u_1, \cdots, u_q$ form a subset. This subset is either a proper subset or a full set of arbitrary unknowns of Σ_2.

Let us assume, then, that this latter condition is satisfied, that is, $u_1, \cdots, u_q$ form a complete set of arbitrary unknowns of both systems Σ_1 and Σ_2.

In order to construct a resolvent of Σ_2 we choose two forms G and Q, with G not in Σ_2 and free of the y_i, such that, for any two distinct solutions of Σ_2 with the same u_i, such that G does not vanish for these solutions, Q yields two distinct functions

* Van der Waerden, *Moderne Algebra*, vol. 2, p. 63.

† Published by this Society, 1932.

‡ We assume that the associated field contains a non-constant function. This involves no loss of generality.