ON THE INFINITE SEQUENCES ARISING IN THE THEORIES OF HARMONIC ANALYSIS, OF INTERPOLATION, AND OF MECHANICAL QUADRATURES*

BY LEOPOLD FEJÉR

1. *Introduction.* The three mathematical theories indicated in my title are so extensive that I should naturally be unwilling within the bounds of a single discussion to give an outline of the totality of the relevant investigations. On the contrary I shall in each case bind myself to a portion of the corresponding theory. The investigations which I have in mind, and which I hope to be able to present to you, have been conducted almost entirely in the twentieth century. Even in this portion of the theory, however, so many brilliant contemporary mathematicians have collaborated that a considerable complex of investigations has resulted. Thus I shall select from this narrower field only a few results,—such, however, as are characteristic and have served as points of departure for further researches.

I shall therefore undertake to give only an outline of these dominant characteristic results, and shall accomplish this by exhibiting as clearly as possible the *single fundamental idea* which unites them. If I can succeed in the course of my lecture in making the investigations of the whole complex seem to you less diversified, I shall have achieved my goal.

2. *Fourier Series.* I begin my exposition with Fourier series. If $f(t)$ denotes an integrable real function of the real variable t, having the period 2π, then the constants

$$(1)\qquad \begin{cases} a_0 = \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} f(t)dt, \\ a_n = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} f(t)\cos nt\,dt, \\ b_n = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} f(t)\sin nt\,dt, \end{cases} \qquad (n = 1, 2, 3, \cdots),$$

* An address delivered by invitation of the authorities of the Century of Progress Exposition and the American Association for the Advancement of Science at the World's Fair in Chicago, June 21, 1933, before the American Mathematical Society and Section A of the A.A.A.S.—The author is indebted to Professor W. A. Hurwitz for the preparation of the English version of this address.