Elementary Mathematics from an Advanced Standpoint. Arithmetic, Algebra, Analysis. By Felix Klein. Translated from the third German edition by E. R. Hedrick and C. A. Noble. New York, Macmillan, 1932. 10+274 pp.

Klein possessed in an unusual degree the abilities of a great mathematician and the gifts of an inspiring teacher and lecturer. He had a broad knowledge of mathematics and a correspondingly deep insight into the foundations and interrelations of its various branches. Both Klein's qualifications for writing a book of this nature and the scarcity of such books combine in directing attention to the present volume.

This book, a translation of the first of Klein's three volumes entitled *Elementarmathematik vom höheren Standpunkte aus*, is a series of lectures that Klein gave for teachers of mathematics in secondary schools. The material is presented under the headings of arithmetic, algebra, analysis, and a supplement. The section on arithmetic treats the extensions of the number system and the laws of operation, beginning with integers and ending with complex numbers and quaternions. The treatment seeks to explain the *how* and *why* of the subject. As an example, we note the discussion of the little understood rule of signs: "minus times minus gives plus." The section on algebra is devoted to the solution of equations. First, some geometric methods are explained for investigating the real roots of rational integral equations containing parameters. Then complex roots are considered, especially of those equations whose solutions lead to a consideration of the groups of motions connected with the regular bodies. Free use is made of Riemann surfaces and other parts of the theory of functions of a complex variable. The section on analysis is devoted to the logarithmic, exponential, and trigonometric functions, and a discussion of the infinitesimal calculus proper. A wide variety of subjects is treated, however, in connection with these general topics: the construction of the early logarithmic and trigonometric tables, expansions in Fourier series, Taylor's Theorem, and Newton's and Lagrange's interpolation formulas will serve as samples. Finally, the supplement contains proofs of the transcendence of e and π, and a discussion of assemblages.

The real excellence of the book, however, is due to certain clearly defined characteristics of the presentation. In the first place, the historical development of the theory is traced. This is not history for history's sake alone, but history as an aid to gaining a deeper insight into the present state of the theory. In this connection it should be stated that the inductive method of presentation is used exclusively.

Secondly, the geometric aspects of the subjects treated are emphasized. It is significant that the book contains 125 figures. The geometric meaning of Fermat's Theorem is explained; the Pythagorean number triples are obtained by a geometric method. The graphs of the approximating polynomials of Taylor's series expansions are drawn in order to show the nature of the convergence and divergence; similarly for Fourier series. Klein would develop geometric intuition and sense perception as an aid to mathematical investigation.

Again, Klein shows the mutual relations between problems in different fields. His ability to discover such relations is well known, and many examples are to be found in this volume. We may mention the parallel treatment of the