BLOCH'S THEOREM FOR MINIMAL SURFACES*

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The following theorem was first proved by Bloch.‡

BLOCH'S THEOREM. *There exists a positive absolute constant* B *with the following property. Let* $Z=f(z)$ *be analytic for* $|z|\leqq 1$, *with* $|f'(0)|\geqq 1$; *then in the Z-plane there is an open circle of radius at least* B, *which is the uniplanar§ map of a portion of the circle* $|z|<1$.

Other proofs of greater simplicity have been given. The present generalization follows the proof given by Landau and by Valiron.|| In this paper we shall prove the following theorem.

THEOREM. *There exists a positive absolute constant* B *with the following property. Let the circle* $u^2+v^2\leqq 1$ *be mapped conformally on a minimal surface, with* $\mathcal{E}_0\geqq 1$, *where* $\mathcal{E}_0$ *denotes the area deformation ratio at the origin; then on the minimal surface there is an open geodesic circle of radius at least* B, *containing no singular points, which is the one-to-one map of a portion of the circle* $u^2+v^2<1$.

That is, there is a point on the surface such that no curve on the surface, issuing from this point and of length less than B, comes either to the boundary of the map or to a point where the conformal character of the map breaks down.

In order that the real analytic functions

$$x_j = x_j(u, v), \qquad (j = 1, 2, 3),$$

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‡ *Les théorèmes de M. Valiron sur les fonctions entières, et la théorie de l'uniformisation*, Comptes Rendus, vol. 178 (1924), pp. 2051–2052, and Annales de la Faculté des Sciences de l'Université de Toulouse, (3), vol. 17 (1925), pp. 1–22.

§ German *schlicht.*

|| Landau, *Über die Blochsche Konstante und zwei verwandte Weltkonstanten*, Mathematische Zeitschrift, vol. 30 (1929), pp. 608–634; Valiron, *Sur le théorème de M. Bloch*, Rendiconti del Circolo Matematico di Palermo, vol. 54 (1930), pp. 76–82.