INVOLUTORIAL LINE TRANSFORMATIONS DETERMINED BY CREMONA PLANE INVOLUTIONS*

BY J. M. CLARKSON

1. *Introduction.* The author has discussed† an involutorial line transformation effected by considering a harmonic homology in each of two planes. If A, B be the points in which an arbitrary line (y) meets the planes α, β, and if A', B' be their images by the homologies I_α, I_β, respectively, then $(x) \equiv A'B'$ is the transform of (y). It is the purpose of the present paper to consider the line transformations similarly determined by Cremona involutorial transformations in each of two planes. All combinations of the four fundamental types: Homology; Jonquières; Geiser; and Bertini will be considered. The orders of the transformations, the invariant loci, the singular elements and the transforms of certain elementary forms are discussed.

2. *Homology-Jonquières.* In the plane α consider a harmonic homology I_α, center at O_1 and axis Δ_α. In the plane β consider the perspective Jonquières involution I_β, of order n, with basis point P_1 of multiplicity $(n-1)$ and basis points $P_2, \cdots, P_{2n-1}$ each simple, and with invariant curve $\Delta_\beta: P_1^{n-2} P_2^1 \cdots P_{2n-1}^1$ of order n and genus $(n-2)$.

An arbitrary line (y) meets α in a point A whose coordinates are linear in the Plücker coordinates y_i of (y) and meets β in a point B whose coordinates are also linear in y_i. The image A' of A by I_α has coordinates also linear in y_i but the image B' of B by I_β has coordinates which are functions of degree n in y_i. Hence $(x) \equiv A'B'$ has Plücker coordinates of degree $(n+1)$ in y_i. Thus, the transformation

$$x_i = \phi_i(y) \tag{1}$$

is of order $(n+1)$. The invariant lines of (1) form a congruence (n, n) composed of the lines meeting Δ_α, Δ_β; and in addition there is a cone of order n, vertex O_1, base curve Δ_β.

* Presented to the Society, April 14, 1933.

† *Some involutorial line transformations*, this Bulletin, vol. 39 (1933), pp. 149–154.