NOTE ON CUBIC SURFACES IN THE GALOIS FIELDS OF ORDER 2^n*

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Let us consider the cubic surface with the equation

$$
(1)\quad \begin{aligned} f(x, y, z, w) \equiv a_0w^3 &+ (b_0x + b_1y + b_2z)w^2 + (c_0x^2 + c_1y^2 \\ &+ c_2z^2 + c_3yz + c_4zx + c_5xy)w + (d_0x^3 \\ &+ d_1y^3 + d_2z^3 + d_3y^2z + d_4yz^2 + d_5x^2y \\ &+ d_6xy^2 + d_7x^2z + d_8xz^2 + d_9xyz) = 0, \end{aligned}
$$

whose coefficients and variables represent numbers in a Galois field of order 2^n. The first polar (or polar quadric) of any point $P'(x', y', z', w')$ with respect to (1) is

$$
(2)\quad \begin{aligned} &(d_0x' + d_5y' + d_7z' + c_0w')x^2 + (d_6x' + d_1y' + d_3z' \\ &+ c_1w')y^2 + (d_8x' + d_4y' + d_2z' + c_2w')z^2 + (b_0x' + b_1y' \\ &+ b_2z' + a_0w')w^2 + (d_9z' + c_5w')xy + (d_9y' + c_4w')xz \\ &+ (c_5y' + c_4z')xw + (d_9x' + c_3w')yz + (c_5x' + c_3z')yw \\ &+ (c_4x' + c_3y')zw = 0. \end{aligned}
$$

The second polar of P' with respect to (1) can be obtained from (2) by interchanging x' and x, y' and y, z' and z, w' and w. The polar quadric of (0, 0, 0, 1) is

$$
(3)\quad c_0x^2 + c_1y^2 + c_2z^2 + a_0w^2 + c_3yz + c_4zx + c_5xy = 0.
$$

The second polar of (0, 0, 0, 1) is

$$
(4)\quad b_0x + b_1y + b_2z + a_0w = 0.
$$

The Hessian of (1) is

$$
(5)\quad \begin{aligned} (d_9z + c_5w)(c_4x + c_3y) &+ (d_9y + c_4w)(c_5x + c_3z) \\ &+ (c_5y + c_4z)(d_9x + c_3w) \equiv 0. \end{aligned}
$$

We note in fact, that the first polar of (0, 0, 0, 1) with respect to (3) vanishes identically; whereas the first polars of (1, 0, 0, 0), (0, 1, 0, 0), and (0, 0, 1, 0), respectively, are

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