ON THE REDUCTION OF A MATRIX TO ITS RATIONAL CANONICAL FORM*

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Two square $n \times n$ matrices A and B with elements in a field F are similar in F if there exists a non-singular $n \times n$ matrix with elements in F such that $S^{-1}AS = B$. In the study of similarity canonical forms play a fundamental role. The classical canonical form for A is one in which the elementary divisors of $A - \lambda I$ are brought into prominence. In 1926, Dickson published in his *Modern Algebraic Theories* a rational discussion of the problem of similarity in which a rational canonical form based on the invariant factors of $(A - \lambda I)$ was used. Other discussions by Lattès, Krull, Kowalewski, and Menge have been published.

The following seems to be, from the algebraic standpoint, a somewhat more direct discussion than others known to the author. Moreover, in arriving at the well known rational canonical form for a matrix, certain lemmas of interest are developed.

Throughout we consider all elements of matrices and vectors and coefficients of polynomials that enter the discussion to be in a field F. All points of interest are met with if the elements involved are rational.

Consider an $n \times n$ matrix A. If the vectors $\xi_1, \xi_2, \cdots, \xi_p$ are $n \times 1$ matrices, we define $L(\xi_1, \xi_2, \cdots, \xi_p)$ to be the linear set consisting of all vectors of the form $\sum_1^p g_i(A)\xi_i$, where the g's are polynomials.

If L is such a linear set and if g is a polynomial and η an $n \times 1$ vector, we say that $g(A)\eta \equiv 0 \bmod L$, where 0 stands for the zero vector if $g(A)\eta$ is in L.

If $g_1(A)\eta \equiv 0 \bmod L$, and $g_2(A)\eta \equiv 0 \bmod L$, then for every pair of polynomials p_1, p_2,

$$\{p_1(A)g_1(A) + p_2(A)g_2(A)\}\eta \equiv 0 \bmod L.$$

Since the greatest common divisor of g_1 and g_2 is expressible in the form $p_1g_1 + p_2g_2$, one can prove (as in the proof for the existence of a minimum equation for a matrix) that for each

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