NOTE ON THE GYROSCOPE

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By a *gyroscope* we mean a rigid body having an axis of material symmetry and free to turn about a certain fixed point O of the axis. Let the gyroscope be acted on by any forces, and let it be required to determine the motion of the axis under the action of these forces. By far and away the most interesting and important case is that in which the component couple tending to produce rotation about the axis is nil, so that the component angular velocity about the axis is constant; denote it by ν. Under this restriction the resultant of the applied forces can be expressed as a single force, $\mathfrak{F}$, acting at a point P on the axis at unit distance from O, normally to OP, and by the force the peg at O exerts.

Let $\mathfrak{C}$ be the curve P describes on the unit sphere about O. The problem is solved by the intrinsic equations:*

$$A\frac{dv}{dt} = T, \qquad A\kappa v^2 + C\nu v = Q,$$

where C and A are the moments of inertia about the axis and about a normal to the axis through O; T and Q are the components of $\mathfrak{F}$ along the tangent and normal (in the tangent plane of the sphere) of $\mathfrak{C}$; and κ is the *bending* of the cone determined by O and $\mathfrak{C}$,—or the rate at which the plane through O tangent to $\mathfrak{C}$ is turning when the point of tangency describes $\mathfrak{C}$ with unit velocity.

The object of the present note is to point out a simple interpretation of these equations in terms of the motion of a material particle carrying a charge of electricity and moving in the electro-magnetic field of force generated by the north pole of a magnet situated at O. Let the mass, m, of the particle be A, or $m = A$; and let the particle be constrained to lie on the unit sphere about O. Let

$$Q_1 = A\kappa v^2, \qquad Q_2 = C\nu v.$$

* See the author's paper, *On the gyroscope*, Transactions of this Society, vol. 23 (1922), p. 240.