# ABSTRACTS OF PAPERS

## SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

123. Professor G. A. Bliss and Dr. M. R. Hestenes: *A note on the problem of Bolza in the calculus of variations.*

In this paper it is shown by a simple reapplication of the first necessary condition on a minimizing arc for the problem of Bolza that for every such arc there exists a function $F=\lambda_0 f+\lambda_\alpha \phi_\alpha$ for which $F-y_i' F_{y_i'}$ and $F_x$ satisfy a Du Bois Reymond equation similar to those satisfied by the functions $F_{y_i'}$ and $F_{y_i}$. These equations are dependent for a minimizing arc which is normal and non-singular. For other arcs no proof has as yet been given that this is so. Consequences of these results are given. It is shown that for a normal minimizing arc the continuity of $F-y_i' F_{y_i'}$ is a consequence of the continuity of $F_{y_i'}$ and the necessary condition of Weierstrass. (Received March 22, 1933.)

124. Dr. M. R. Hestenes: *Sufficient conditions for the problem of Lagrange in the calculus of variations.*

In this paper a set of sufficient conditions for the problem of Lagrange with fixed end points is given in which it is assumed that the arc $E_{12}$ under consideration is normal on the interval $x_1 x_2$ but no assumption is made regarding normality on the sub-intervals of $E_{12}$. This result is attained by replacing the usual condition that $E_{12}$ contains no point conjugate to the point 1 by a new condition which states that a certain quadratic form is positive on the interval $x_1 x_2$ and positive definite at one value of $x$ on $x_1 x_2$. This new condition is a consequence of the theory of broken extremals applied to the problem of the second variation. It is closely related to a condition given by Bliss for variable end point problems in the plane and also the fundamental quadratic form of Morse. The relations between the sufficient conditions here given and known sufficient conditions are discussed. In particular an example is given in which the minimizing arc $E_{12}$ satisfies the conditions of this paper but which is not normal on every sub-interval $x_1 x_3$ nor on every subinterval $x_3 x_2$ and hence does not satisfy the sufficient conditions given heretofore. (Received April 8, 1933.)

125. Professor Oswald Veblen: *Geometry of two-component spinors.*

This paper shows how a theory of two-component spinors can be derived from the geometry of the light-cones in the tangent spaces of the space-time