involves the operation of folding and is important in the theory of almost periodic functions. The main theorem which is finally proved at the end of the lectures is that the class of almost periodic functions is identical with the class $H\{s(x)\}$ of functions which can be uniformly approximated by finite sums $s(x)$ composed of terms of type $a_n \exp(i\lambda_n x)$, where the coefficients a_n are complex quantities and the exponents λ_n real quantities all of which can be chosen freely. The analysis deals largely with mean value theorems, the multiplication theorem, the uniqueness theorem and its equivalence to the Parseval relation, limiting forms of integrals, and a convergence theorem for infinite series of type $s(x)$ when the Fourier exponents are linearly independent. The report closes with an account of some generalizations of the idea of an almost periodic function. The work is all of a high standard and will stand the scrutiny of mathematicians who insist on a statement of all the saving restrictions.

H. BATEMAN

KAMKE ON PROBABILITY

Einführung in die Wahrscheinlichkeitstheorie. By Erich Kamke. Leipzig, S. Hirzel, 1932. vii+182 pp.

This excellent *Theory of Probability* is an elaboration of the author's paper *Über neuere Begründungen der Wahrscheinlichkeitsrechnung.** In this paper Kamke sets forth a new foundation for probability and gives a critique of some closely related theories, those of R. von Mises, Dörge, and Tornier. Von Mises—whose work marks the beginning of a new era in probability theory—discards the conception of "equal likely events" as a proper basis for probability, in favor of a treatment based upon infinite sequences. If a die is thrown, and the results recorded,—for example, 2, 5, 3, 3, 6, 1, 4,—a sequence is obtained which may be thought of as continuing indefinitely. Now let m be the number of times that the five-spot appears in the first n throws. Some authors naively write: $\lim m/n = 1/6$. But, in general, no such limit exists. We know nothing about the future behavior of the die. A rigorous treatment of probability must divorce itself from all physical considerations. This the newer theories do. We may write: $a_1, a_2, \cdots, a_n, \cdots$, as symbols for a set of numbers, restricted to the values 1, 2, 3, 4, 5, and 6. And among all possible sequences of this description we may *choose to consider only* those for which $\lim m/n = 1/6$. And the properties of such sequences may be ascertained with mathematical rigor. Sometimes the sequence definition is called the "statistical" definition of probability. This is unfortunate, for the word "statistical" is likely to carry a reader's mind to the loose "limit" first mentioned. A sharp distinction between such a *pseudo-limit* and a real limit is fundamental for all clear thinking in probability. The theory of R. von Mises is grounded upon two axioms; that is, he *considers* only those sequences for which both axioms are valid. The first axiom relates to the existence of limits, such as has just been illustrated. The second axiom involves a selection—from the original sequence—of terms by some scheme which relates to their position or order—to the subscript r of a_r—

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