

HAUSDORFF TRANSFORMATIONS FOR DOUBLE SEQUENCES

BY C. R. ADAMS

1. *Introduction.* The purpose of this note is to extend to double sequences some of the results of Hausdorff's notable papers on methods of summability and moment-sequences.†

Let $\lambda = \|\lambda_{pqmn}\|$, $(p, q, m, n = 0, 1, 2, \dots)$, be a four-dimensional matrix of real or complex numbers. Then the system of equations

$$(1) \quad A_{pq} = \sum_{m,n=0}^{\infty} \lambda_{pqmn} a_{mn},$$

if the series all converge, transforms a double sequence $\{a_{mn}\}$ into a new double sequence $\{A_{pq}\}$. Necessary and sufficient conditions‡ that this transformation be convergence-preserving for bounded sequences are the following:

$$(A) \quad \sum_{m,n=0}^{\infty} |\lambda_{pqmn}| \leq M, \quad (p, q = 0, 1, 2, \dots),$$

$$(B) \quad \lim_{p,q \rightarrow \infty} \sum_{m,n=0}^{\infty} \lambda_{pqmn} = l,$$

$$(C) \quad \lim_{p,q \rightarrow \infty} \lambda_{pqmn} = l_{mn}, \quad (m, n = 0, 1, 2, \dots),$$

$$(D) \quad \lim_{p,q \rightarrow \infty} \sum_{m=0}^{\infty} |\lambda_{pqmn} - l_{mn}| = 0, \quad (n = 0, 1, 2, \dots),$$

† Hausdorff, *Summationsmethoden und Momentfolgen*, I, II, *Mathematische Zeitschrift*, vol. 9 (1921), pp. 74–109, 280–299.

‡ See Robison, *Divergent double sequences and series*, *Transactions of this Society*, vol. 28 (1926), pp. 50–73, especially pp. 71–72. Such transformations are sometimes, if not always, convergence-preserving for certain unbounded sequences; see Adams, *Transformations of double sequences, with application to Cesàro summability of double series*, this *Bulletin*, vol. 37 (1931), pp. 741–748; Löscher, *Über den Permanenzsatz gewisser Limitierungsverfahren für Doppelfolgen*, *Mathematische Zeitschrift*, vol. 34 (1931), pp. 281–290; Adams, *On summability of double series*, *Transactions of this Society*, vol. 34 (1932), pp. 215–230, hereafter referred to as *A*; and Agnew, *On summability of double sequences*, *American Journal of Mathematics*, vol. 54 (1932), pp. 648–656.