SOLUTION OF HUNTINGTON'S "UNSOLVED PROBLEM IN BOOLEAN ALGEBRA"

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The sixth set of postulates for Boolean algebra recently proposed by E. V. Huntington* may, as he suggests, be modified so as to read as follows. Let

K = an undefined class containing at least two elements, $a, b, c, \cdots$;
T = an undefined subclass in K (so that if a is in T, then a is in K);
$(a+b)$ = the result of an undefined binary operation; and
a' = the result of an undefined unary operation.

POSTULATE 1.71. If a and b are in K, then $(a+b)$ is in K.
POSTULATE 1.7. If a is in K, then a' is in K.
POSTULATE 1.1. If a is in T and $(a'+b)$ is in T, then b is in T.
POSTULATE 1.2. If a is in K, then $[(a+a)'+a]$ is in T.
POSTULATE 1.3. If a, b, etc. are in K, then $[b'+(a+b)]$ is in T.
POSTULATE 1.4. If a, b, etc. are in K, then $[(a+b)'+(b+a)]$ is in T.
POSTULATE 1.6. If a, b, c, etc. are in K, then $\{(b'+c)'+[(a+b)'+(a+c)]\}$ is in T.
POSTULATE 1.8. If $(a'+b)$ is in T and $(b'+a)$ is in T, then $a=b$.

The "unsolved problem" he proposes is the question whether or not Postulate 1.1 is independent of the other postulates in this list. The purpose of the present paper is to answer this question

* E. V. Huntington, *New sets of independent postulates for the algebra of logic, with special reference to Whitehead and Russell's Principia Mathematica*, Transactions of this Society, vol. 35 (1933), pp. 274–304, especially p. 298. Huntington's sixth set, while inferior to his fourth set when regarded merely as a set of postulates for Boolean algebra, is of interest in connection with B. A. Bernstein's version of the primitive propositions of the *Principia* (see the bibliography in the paper cited). In connection with Huntington's fourth set, it should be noted that Postulate 4.5 is not independent; see the forthcoming number of the Transactions of this Society.