LINEAR INTEGRAL EQUATIONS OF FUNCTIONS OF TWO VARIABLES*

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1. *Introduction.* It is the purpose of this paper to consider certain conditions for the solution of the following linear integral equation:

$$\bar{y}(\alpha,\beta) = y(\alpha,\beta) + \lambda\int_a^b K(\alpha,\sigma)y(\sigma,\beta)d\sigma + \mu\int_a^b L(\beta,\tau)y(\alpha,\tau)d\tau$$

$$+ \nu\int_a^b\int_a^b M(\alpha,\beta,\sigma,\tau)y(\sigma,\tau)d\sigma d\tau$$

and, particularly, the truncated form with $M(\alpha,\beta,\sigma,\tau)\equiv 0$. The more important results of the paper are to be found summarized in Theorems 2 and 3.

Throughout the paper we shall consider all given functions as bounded and continuous, and in order to facilitate the work we shall adhere to the notation (1) *to represent the variables of functions as indices*, (2) *to signify by the repetition of an index in a term, once as a subscript and once as a superscript, an integration on that variable over the fundamental interval* (a, b).

2. *A Generalization of the Fredholm Equation.* Let us consider a special type of integral equation of a function of two variables which has as its origin the succession of two ordinary Fredholm equations, namely

$$(1)\qquad \bar{y}^{\alpha\beta} = y^{\alpha\beta} + \lambda K_\sigma^\alpha y^{\sigma\beta} + \mu L_\tau^\beta y^{\alpha\tau} + \lambda\mu K_\sigma^\alpha L_\tau^\beta y^{\sigma\tau}.$$

In fact, (1) is given by the succession of equations

$$(2)\qquad z^{\alpha\beta} = y^{\alpha\beta} + \lambda K_\sigma^\alpha y^{\sigma\beta},\quad \bar{y}^{\alpha\beta} = z^{\alpha\beta} + \mu L_\tau{}^\beta z^{\alpha\tau}.$$

The equations (2) being ordinary Fredholm equations, it is evident at once that the equation (1) has the unique, continuous inverse

$$(3)\qquad y^{\alpha\beta} = \bar{y}^{\alpha\beta} + \lambda k_\sigma^\alpha \bar{y}^{\sigma\beta} + \mu l_\tau^\beta \bar{y}^{\alpha\tau} + \lambda\mu k_\sigma^\alpha l_\tau^\beta \bar{y}^{\sigma\tau},$$

providing that λ and μ are not characteristic values of their re-

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