THE PROBLEM OF PLATEAU†

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1. *Formulation.* The problem of Plateau is to prove the existence of a minimal surface bounded by a given contour. The first and only complete solution of this problem was found by the present author [1–11]‡; in this solution the contour is an arbitrary Jordan curve in n-dimensional euclidean space. Besides this generality of result, the chief contribution of the work seems to lie in the introduction of a certain new functional $A(g)$, which furnishes the key to the Plateau problem and makes clear its relationship to other fundamental problems of analysis.

For definition of minimal surface, we take the formulas of Weierstrass,

$$x_i = \Re F_i(w), \quad \sum_{i=1}^{n} F_i'^2(w) = 0, \tag{1}$$

though Weierstrass himself considered only the case $n=3$. Here w is a complex variable, $w=u+iv$, of which the F_i are monogenic functions. When $n=2$, these formulas become

$$x_1 = \Re F_1(w), \; x_2 = \Re F_2(w), \; F_1'^2(w) + F_2'^2(w) = 0, \tag{2}$$

or

$$F_2'(w) \pm iF_1'(w) = 0. \tag{3}$$

If

$$F_1(w) = P_1(u, v) + iQ_1(u, v), \qquad F_2(w) = P_2(u, v) + iQ_2(u, v),$$

then

$$F_1'(w) = \frac{\partial P_1}{\partial u} + i\frac{\partial Q_1}{\partial u} = \frac{\partial P_1}{\partial u} - i\frac{\partial P_1}{\partial v},$$

$$F_2'(w) = \frac{\partial P_2}{\partial u} + i\frac{\partial Q_2}{\partial u} = \frac{\partial P_2}{\partial u} - i\frac{\partial P_2}{\partial v},$$

† Address delivered by invitation of the program committee at the New York meeting, October 29, 1932.

‡ Numerical references are to the bibliography at the end.