ON SINGULAR CHAINS AND CYCLES

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1. *Introduction.* The theory of the topological invariance of the absolute or relative combinatorial characters of a complex, as developed in our Colloquium Lectures on *Topology* (Chapter II), was based, following Alexander and Veblen, upon the concept of singular chain. Our presentation, and indeed any known to us, appears to give rise to many misconceptions which it is proposed to clear up in the present note. Unless otherwise stated the notations are those of *Topology.*

2. *Singular Cells.* Let $\mathfrak{R}$ be a topological space and let e_p be a simplicial oriented cell such that there exists a continuous single-valued transformation (=c.s.v.t.) T of the point set e_p into a subset E_p of $\mathfrak{R}$, where $E_p = Te_p$. The symbol (e_p, T, E_p), associated with the set $\overline{E}_p$ is called a *singular oriented p-cell on* $\mathfrak{R}$. If e_p' is another e_p, there exists a barycentric transformation U of $\bar{e}_p'$ into $\bar{e}_p$: $U\bar{e}_p' = \bar{e}_p$. If we set $T' = TU$, it is evident that (e_p', T', E_p) defines also a singular oriented p-cell on $\mathfrak{R}$. We shall agree to consider it as identical with the first:

$$(1) \qquad (e_p', T', E_p) = (e_p, T, E_p).$$

This has the advantage of freeing the notion of singular cell from a too narrow connection with a specific image e_p.

3. *Singular Chains.* The singular p-chain C_p on $\mathfrak{R}$ is now defined as the association of a symbol

$$(2) \qquad C_p = \sum t_i(e_p^i, T^i, E_p^i)$$

with coefficients t belonging to one of the three rings (rational numbers, integers, integers mod m) considered in *Topology*, together with the set of all sets $\overline{E}_p^i$ corresponding to t's$\neq 0$. As a special case the e's might be cells of a finite complex k such that there exists a c.s.v.t. T of k into a subset of $\mathfrak{R}$. Then the chain symbol may take the form

$$(3) \qquad C_p = \sum t_i(e_p{}^i, T, E_p^i),$$

and C_p may be considered as the image of the subchain