ON THE CLASS NUMBERS OF A CYCLIC FIELD AND A SUB-FIELD*

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1. *Introduction.* Let F_1 be an algebraic field which is cyclic with respect to the rational field and let F_2 be a sub-field of F_1. Kummer stated that if F_1 is a divisor of the field defined by a λth root of unity, λ a prime, then the class number of F_2 is a divisor of the class number of F_1.† He employed the broader definition of equivalence. However, as pointed out by Hilbert,‡ there is an error in his proof. Furtwängler proved the theorem for the case where λ is a power of a prime, using narrow equivalence.§

The purpose of this paper is to prove the following theorem which overlaps but does not include Furtwängler's.

THEOREM. *Let F_1 be a field which is cyclic with respect to the rational field and such that the discriminant of every sub-field, not rational, contains a prime factor not a divisor of the degree F_1. If F_2 is a sub-field of F_1 and if h_1, h_2 are the number of classes of narrowly equivalent ideals in F_1, F_2, respectively, then h_2 is a divisor of h_1.*

Furtwängler gave an example of a non-cyclic abelian field F_1, for which this theorem is not valid.||

2. *A Lemma.* In the next paragraph, the above theorem will be proved by use of the following lemma and a theorem due to Chevally.

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† Journal für Mathematik, vol. 40 (1850), pp. 114–6; Bulletin of the National Research Council, No. 62, *Algebraic Numbers*, II, Vandiver and Wahlin, p. 16.

‡ *Bericht über die Theorie der algebraischen Zahlkörper*, p. 378.

§ Journal für Mathematik, vol. 134 (1908), pp. 91–94. In this article, Furtwängler states (p. 91) that Kummer's theorem is correct since his result is a generalization of Kummer's. Since he and Kummer used different definitions of equivalence, it is not obvious that his theorem includes Kummer's and the validity of the latter is still an open question.

|| Loc. cit., p. 94.