ON THE EXISTENCE OF THE ABSOLUTE MINIMUM IN PROBLEMS OF LAGRANGE†

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In a previous paper,‡ I remarked that the theorems obtained are applicable to problems of Lagrange in which the side conditions are of the form

$$g_\mu(x, y_1, \cdots, y_k) = 0, \qquad (\mu = 1, \cdots, m). \tag{1}$$

It is easily seen also that *the proofs* there made are applicable under suitably *weakened hypotheses* (to be stated below) to problems of Lagrange in which appear side conditions which are *differential equations* of the form

$$y_\sigma' = h_\sigma(x, y_1, \cdots, y_k), \quad (\sigma = 1, \cdots, s). \tag{2}$$

Calculus of variations problems in which the integrand involves higher derivatives are reducible to Lagrange problems of this type, as are also isoperimetric problems in which the integrands of the integrals to be kept constant depend only on the coordinates. Examples in which the stronger hypotheses of the paper cited do not hold are given in the final paragraph.

We suppose that the integrand function $f(x, y_1, \cdots, y_k, y_1', \cdots, y_k')$ and the functions g_μ, h_σ, together with the partial derivatives $f_{y_i'}$, $g_{\mu x}$, $g_{\mu y_i}$, are defined and continuous for all points (x, y) in a closed domain A and for all y'. Let R^* denote the set of all points (x, y, y') having (x, y) in A and satisfying equations (1), (2) and

$$g_{\mu x} + g_{\mu y_i} y_i' = 0, \qquad (\mu = 1, \cdots, m). \tag{3}$$

An *admissible curve* C, $y_i = y_i(x)$, is one which is absolutely continuous and has all its elements (x, y, y') in R^*.§ Then if K is a closed class of absolutely continuous curves, the sub-class K^*

† Presented to the Society, December 27, 1932.

‡ *On the existence of the absolute minimum in space problems of the calculus of variations*, Annals of Mathematics, vol. 28 (1927), pp. 153–170.

§ The set of points x at which one or more of the functions y_i fails to have a finite derivative may be neglected.