SETS OF LOCAL SEPARATING POINTS OF A CONTINUUM*

BY G. T. WHYBURN

1. *Introduction.* Let M denote any locally compact metric continuum and let L be the set of all local separating points† of M. We proceed to establish the following six properties, of which, for our immediate purposes, the most useful is number (iv).

(i). *If U is any uncountable subset of L, there exists a point x of U which is a point of order 2 in M relative to U.*

This statement means that x is contained in arbitrarily small neighborhoods whose boundaries have in common with M just two points and these two points belong to U. A proof has already been given by the author (loc. cit.).

(ii). *If H is any connected subset of M, then $(\overline{H}-H)\cdot L$ is countable.*

For if not, (i) would give a point x of this set which could be separated in M from some point of H by two points not in H, which obviously is impossible since $H+x$ is connected.

(iii). *If H is any connected subset of M, the points of $H\cdot L$ which are not local separating points of H are countable.*

This results immediately from (i).

(iv). *If H is any connected subset of M such that $\overline{H}\subset L+C$, where C is some countable set, then H is a locally connected G_δ-set. Hence H is arcwise connected.*

By (ii) we see that $(\overline{H}-H)\cdot L$ and hence $\overline{H}-H$ itself is countable. Thus H is a G_δ-set. Now $\overline{H}$ must be a regular curve, for by (i), all save a countable number of its points are points of order 2. Thus any connected subset of $\overline{H}$, and in particular H, is locally connected. That H is arcwise connected follows now by the well known theorem of Moore-Menger.‡

---

* Presented to the Society, February 25, 1933.

† A point p is a local separating point of M provided some neighborhood V of p exists such that $M\cdot\overline{V}-p$ is separated between some pair of points belonging to the component of $M\cdot\overline{V}$ which contains p. See the author's paper in Monatshefte für Mathematik und Physik, vol. 36 (1929), pp. 305–314.

‡ See R. L. Moore, *Foundations of Point Set Theory*, Colloquium Publications of this Society, vol. 13 (1932), p. 86; and K. Menger, Monatshefte für Mathematik und Physik, vol. 36 (1929), pp. 193–218.