NON-ANALYTIC FUNCTIONS OF A COMPLEX VARIABLE*

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1. *Introduction.* The outlines of the theory of non-analytic functions of a complex variable, called also polygenic functions, have been stated in recent years in a number of articles.† Indeed, from the very first of the modern study of the theory of functions, going back at least as far as the famous inaugural dissertation of Riemann, the beginnings of the subject have been mentioned essentially, if for no other purpose than to state the conditions under which a function of a complex variable is analytic, and to delimit the field of functions to be studied.

In the present address, such preliminary details will be mentioned only briefly, with references; but enough of them must be stated to develop a notation, and to give the proper setting. More detailed attention will be given to those developments which have taken place during the last decade, and to some hitherto unpublished facts. A brief review of some of the historical background will serve both its obvious purpose, and also that of introducing the necessary preliminary details and notations.

2. *Historical Background.* As was stated above, every careful presentation of the classical theory of functions of a complex variable did include in a measure the elementary ideas for the general case of *any* function of a complex variable. A function

$$w = f(z) = \phi(x, y) + i\psi(x, y), \tag{1}$$

where $w = u + iv$ and $z = x + iy$, is said to be defined for a given region (or set of values) of z if w is determined whenever z is assigned a value in that region (or set). The equation (1) is then equivalent, of course, to the two real simultaneous equations

$$u = \phi(x, y), \quad v = \psi(x, y), \tag{2}$$

which themselves express a transformation of the xy plane onto

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† See the list of recent articles at the end of this paper.