SHORTER NOTICES

Principes Géométriques d'Analyse, Deuxième Partie. By Gaston Julia. Paris, Gauthier-Villars, 1932. vii+121 pp.

This book presents a continuation of the first volume of the author dealing with those aspects of the modern theory of functions of a complex variable which are derivable from simple geometrical principles. As the author himself points out in the preface to the first volume, the most important of these principles is the conformal correspondence between two regions of planar character or two Riemann surfaces realized by an analytic function $Z=f(z)$. The first volume is principally devoted to the study of those conformal correspondences which carry a unit surface into a Riemann surface interior to the unit circle. This subject centers about the Lemma of Schwarz and the interpretation of it in terms of the non-euclidean (Lobachevskian) geometry in a circle, indicated for the first time by G. Pick.

The second volume begins with the maximum principle and consists of generalizations of this principle in various directions associated with the names of Lindelöf, Littlewood, Carleman, and others. By a series of easy extensions from the maximum principle the author arrives toward the end of Chapter 1 at the Phragmén-Lindelöf theorem. Some applications are then made to the theory of integral functions. The chapter ends with an important Lemma of Carleman concerning analytic functions $f(z)$ in regions $AOBC$ where $\overline{OA}$ and $\overline{OB}$ are rectilinear segments and ACB is a Jordan arc. The lemma gives a limitation of $|f(z)|$ on the bisector of the angle AOB when the upper bound of $|f(z)|$ on $\overline{OA}$, $\overline{OB}$, and on the arc ACB, and the magnitude of the angle AOB are known.

Chapter 2 is devoted to some classical facts concerning harmonic functions of two variables and the conformal mapping of simply connected regions on a circle. It is rather unfortunate that in connection with the second topic no reference is made to the important work of W. F. Osgood. The author is further led to consider the problem of Dirichlet and in particular the integral of Poisson. One of the methods described for obtaining the integral is that of reducing it to the mean value theorem for harmonic functions. Here again the reference is not wholly adequate since no mention is made of Bôcher, the first to notice the fact. With the mathematical tools thus obtained, the author proceeds to generalize the lemma of Carleman by giving a limitation of $|f(z)|$ valid in every point of the region $AOBC$.

After a brief introduction to the elliptic modular function in Chapter 3, an exposition of the method of Lindelöf follows in Chapter 4. Here the geometric concept underlying the whole work is a relation between two Riemann surfaces S and S_1 which Julia characterizes by saying that S_1 is carried by S. This means that S_1 is a covering surface of S such that every closed curve on S_1 is projected on a closed curve on S. From Lindelöf's fundamental theorem, which contains the lemma of Schwarz as a particular case, a variety of inequalities is developed for functions analytic in a circle. These inequalities, together with