ON THE APPROXIMATE SOLUTION OF LINEAR DIFFERENTIAL EQUATIONS WITH BOUNDARY CONDITIONS*

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In a recent paper† the writer has studied the convergence of trigonometric and polynomial approximating sums to the solution of the mth order linear differential system

$$(1)\quad \begin{aligned} L(y) &\equiv \frac{d^m y}{dx^m} + Q_1(x)\frac{d^{m-1}y}{dx^{m-1}} + \cdots + Q_m(x)y = R(x), \\ U_i(y) &\equiv \sum_{j=1}^{m}\left\{\alpha_i^{(j-1)}y^{(j-1)}(a) + \beta_i^{(j-1)}y^{(j-1)}(b)\right\} = h_i, \\ &\qquad\qquad (i = 1, 2, \cdots, m), \end{aligned}$$

the approximating sums being defined by a least rth power criterion of best approximation. Thus in the polynomial case the approximating sum $P_n(x)$ was defined to be a polynomial of the nth degree which satisfies the boundary conditions $U_i(P_n) = h_i$ and at the same time minimizes the integral $\int_a^b |L(P_n) - R|^r dx$ in comparison with all other polynomials of that type, r being a preassigned constant >0.

The purpose of this paper is to discuss the convergence question when a different criterion is used to define $P_n(x)$, namely, $P_n(x)$ is *the approximating polynomial* of the nth degree for the solution of the system (1) if it minimizes the expression

$$(2)\quad \int_a^b |L(P_n) - R|^r dx + \sum_{i=1}^{m} C_i |U_i(P_n) - h_i|^{r_i}$$

in comparison with all other polynomials of like degree, the r's and C's being given constants >0. Kryloff‡ and Picone§ have

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† W. H. McEwen, *Problems of closest approximation connected with the solution of linear differential equations*, Transactions of this Society, vol. 33 (1931), pp. 979–997.

‡ N. Kryloff, *Les Problèmes Fondamentaux de la Physique Mathématique et de la Science d'Ingénieur*, Monographie dans le Domaine de la Mathématique Appliquée, 1932, pp. 234–240.

§ M. Picone, *Sul metodo delle minime potenze ponderate e sul metodo di Ritz*, etc., Rendiconti di Palermo, vol. 52 (1928), pp. 237–244.