PERIODIC SOLUTIONS OF LINEAR DIFFERENTIAL EQUATIONS

BY W. B. FITE

The present article contains a generalization and a correction of certain results given by the writer in an earlier article under the same title.* Consider the equation

$$L(y) \equiv p_0 y^{(n)} + p_1 y^{(n-1)} + \cdots + p_n y = 0, \tag{1}$$

in which the coefficients are one-valued and of period l, and $p_0 \neq 0$ for any real value of x. If $y_1, y_2, y_3, \cdots, y_n$ form a funda mental system of solutions, then

$$D \equiv \begin{vmatrix} y_1 & y_1' & \cdots & y_1^{(n-1)} \\ y_2 & y_2' & \cdots & y_2^{(n-1)} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ y_n & y_n' & \cdots & y_n^{(n-1)} \end{vmatrix} = D_0 e^{I(a,x)},$$

where

$$I(a, x) = -\int_a^x \frac{p_1}{p_0} dx,$$

and where $D_0(\neq 0)$ is the value of D for $x = a$. Since the coefficients of (1) are of period l, $y_i(x+l)$, $(i = 1, 2, \cdots, n)$, form a fundamental system of solutions, and therefore

$$y_i(x + l) = \sum_{j=1}^{n} a_{ij} y_j. \tag{2}$$

The characteristic equation of the substitution (2) is

$$\begin{vmatrix} a_{11} - \omega & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \omega & \cdots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \cdots & a_{nn} - \omega \end{vmatrix} = 0. \tag{3}$$

* Fite, Annals of Mathematics, (2), vol. 28 (1926), pp. 59–64.