ON n-WEBS OF CURVES IN A PLANE

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This note contains a proof of Theorem 4 of the list given by W. Blaschke* in a preceding paper.

If t_i = const. represents n sheaves of curves in a plane, then the maximal number of linearly independent relations

$$(1) \qquad \sum_i U_{ik}(t_i) = 0, \quad (k = 1, \cdots, m, i = 1, \cdots, n),$$

is

$$(2) \qquad N = \tfrac{1}{2}(n-1)(n-2).$$

Let (1) be any set of such relations; then we consider $U_{ik}(t_i)$, $(k=1, \cdots, m)$, for a fixed i to be the m coordinates of a point describing a curve $p_i(t_i)$ in an affine m-space.

If we can prove that the curves $p_i(t_i)$ all lie in parallel linear subspaces of dimension N, our theorem is proved, for this means that between the coordinates of every p_i there exist linear relations with the same constant coefficients, which express $m-N$ of the coordinates in terms of the other N. And this means that of the m relations (1) there can be only N linearly independent.

If we assume our functions U_{ik} to be differentiable a suitable number of times, however, this last statement comes down to proving that among the vectors

$$(3) \qquad \frac{d}{dt_i} p_i(t_i) = p_i'(t_i), \quad p_i''(t_i), \quad p_i'''(t_i), \cdots,$$

there cannot be more than N linearly independent ones.†

We will prove this for $n=5$, $N=6$; the proof can easily be extended to all values of n. To avoid the use of many indices, we will write (1) in the form

$$(4) \qquad p_1(u) + p_2(v) + p_3(r) + p_4(s) + p_5(t) = 0.$$

* W. Blaschke, *Results and problems about n-webs of curves in a plane*, this Bulletin, vol. 38 (1932), p. 828.

† This does not really make it necessary to assume the functions (1) to be analytic; from a certain order m we can always replace (3) by an existence statement for solutions of a differential equation.