A PROPERTY RELATED TO COMPLETENESS*

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In 1926, R. L. Moore presented the following axiom.

AXIOM 1′. *There exists a countable sequence* $G_1, G_2, G_3, \cdots$ *such that* (a) *for each* n, G_n *is a collection of domains covering space,* (b) *if* P_1 *and* P_2 *are distinct points of a domain* R, *there exists an integer* d *such that if* $n > d$ *and* K_n *is a domain containing* P_1 *and belonging to* G_n, *then* $\overline{K}_n$ *is a subset of* $R - P_2$, *and* (c) *if* $R_1, R_2, R_3, \cdots$ *is a sequence of domains such that, for each* n, R_n *belongs to* G_n *and such that, for each* n, $R_1, R_2, \cdots, R_n$ *have a point in common, then there exists a point common to all the point sets* $\overline{R}_1, \overline{R}_2, \overline{R}_3, \cdots$.†

Moore has given an example of a non-metric space in which his axiom 1′ holds true. He raised the question as to whether or not a metric space in which his axiom 1′ holds true is *complete*.‡ The present paper answers this question in the affirmative.§

THEOREM. *A metric space* S *in which axiom* 1′ *holds true is complete.*

PROOF. Let $\delta(x, y)$ be a distance function defined over the space S. Let P be any point of S and let n be any positive integer. Either (1) there is a domain of the set G_n which contains every point y such that $\delta(P, y) \leqq 2$, or (2) there exists a greatest number k $(k \leqq 2)$ such that if $r < k$, then there exists a domain of the set G_n containing every point y such that $\delta(P, y) \leqq r$. Let

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‡ A space S is said to be *complete* if there exists a definition of distance such that every sequence of points satisfying the Cauchy condition has a limit point. A sequence of points $P_1, P_2, \cdots$, in a metric space is said to satisfy the Cauchy condition with respect to the distance function δ if, for every positive number e, there exists an integer n such that $\delta(P_n, P_k) < e$ if $k > n$.

§ The present result was obtained about September 1, 1930, and was reported to Professor Moore at that time. I purposely delayed publishing the paper in order that it might not appear in advance of the publication of his book *Foundations of Point Set Theory*. Later in the fall of 1930 Leo Zippin obtained a theorem which, with *other theorems in the literature*, yields the result of this paper.