ON THE RELATIONSHIP AMONG THE DIAGONAL FILES OF A PADÉ TABLE*

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1. *Introduction.* The object of the following note is to investigate the relationship among the nth approximants of the different diagonal files of a Padé table; and to study the relationship among the limits of those files for a Stieltjes power series, in the case † that those files have different limits. We have found that an arbitrary file S_k converges to an expression of the form

$$\frac{\alpha_k p - \beta_k p_1}{\alpha_k q - \beta_k q_1}, \tag{1}$$

where p, p_1, q, q_1 are entire transcendental functions independent of k, and α_k, β_k are polynomials or constants. If we denote by u_k, v_k the numerator and denominator, respectively, of (1), then if k', k'' are two values of the index k, the following identity obtains:

$$u_{k'}v_{k''} - u_{k''}v_{k'} = \alpha_{k'}\beta_{k''} - \alpha_{k''}\beta_{k'}; \tag{2}$$

and the polynomial on the right is not identically zero if $k' \neq k''$.

2. *Preliminary Formulas.*‡ Let $\mathfrak{P}(x) = \sum_{v=0}^{\infty} c_v(-x)^v$ be a *normal* power series, and let $\mathfrak{C}(x) = \sum_{v=0}^{\infty} d_v(-x)^v$ be the reciprocal of $\mathfrak{P}(x)$. Set $\mathfrak{P}^{(k)}(x) = \sum_{v=0}^{\infty} c_{v+k}(-x)^v$, $\mathfrak{C}^{(k)}(x) = \sum_{v=0}^{\infty} d_{v+k}(-x)^v$, $k = 0, 1, 2, \cdots$. Then the series $\mathfrak{P}^{(k)}(x)$, $\mathfrak{C}^{(k)}(x)$ have corresponding continued fractions

$$\frac{1}{a_1^{(k)}} + \frac{x}{a_2^{(k)}} + \frac{x}{a_3^{(k)}} + \cdots, \quad \frac{1}{b_1^{(k)}} + \frac{x}{b_2^{(k)}} + \frac{x}{b_3^{(k)}} + \cdots,$$

respectively, where the numbers $a_n^{(k)}$, $b_n^{(k)}$ are different from 0.

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† Designated as "Case I" in the writer's paper, *On the Padé approximants associated with the continued fraction and series of Stieltjes*, Transactions of this Society, vol. 31 (1929), pp. 91–116. We show in the present article that *no two* of the diagonal files have the same limit, thus supplementing the earlier result.

‡ For details concerning the statements in this paragraph, see a paper by the writer in the Transactions of this Society, vol. 33 (1931), pp. 511–532.