A NOTE ON NORMAL DIVISION ALGEBRAS OF ORDER SIXTEEN*

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1. *Introduction.* I have proved† that every normal division algebra of order sixteen over any non-modular field F contains a quartic field with G_4 group. This important result gave a *determination of all normal division algebras of order sixteen.* I have recently proved‡ the existence of *non-cyclic* normal division algebras, so that the result mentioned above is actually the best possible result. However, my proof of 1929 is long and complicated and the above result there obtained is of sufficient importance to make a better proof desirable. It is the purpose of this note to provide such a proof.

2. *Results Presupposed.*§ We shall require certain well known results on normal division algebras D of order sixteen over F. Algebra D has rank four, so that every sub-field of D is either a quartic field, a quadratic field, or F itself. We also have the following theorems.

THEOREM 1. *Every root in D of the minimum equation of a quantity x of D is a transform yxy^{-1} of x by y in D.*

THEOREM 2. *If $\phi(\omega)=0$ is the minimum equation of x in D, then there exist quantities $x_i=x_1, x_2, \cdots, x_r$ in D such that*

$$\phi(\omega) \equiv (\omega - x_r) \cdots (\omega - x_2)(\omega - x_1).$$

It is obvious that no two adjacent transforms x_{i+1}, x_i in the above irreducible equation can be equal.

From the well known Wedderburn theorem on linear associative algebras with normal simple sub-algebras we have immediately the following result.

* Presented to the Society, August 31, 1932.

† Transactions of this Society, vol. 31 (1929), pp. 253–260.

‡ See this Bulletin of June 1932 and the Transactions of this Society of January 1933 for proof.

§ See my paper, Annals of Mathematics, vol. 30 (1929), pp. 322–338, for a proof of Theorem 1 and a reference to proofs of Theorem 2 and the Wedderburn result implying Theorem 3.