QUADRATIC PARTITIONS: PAPER IV

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1. *Identity of Degree* 5. For the preceding note, see this Bulletin, vol. 38, p. 569. The *degree* of a ϑ, ϕ identity is the degree of the identity in functions ϑ, ϕ. In II we discussed an identity of degree 4. Here we consider an identity of degree 5 whose equivalent in parity functions refers to $F(w, z, u, v)$ as in II. The identity is one of many of degree 5 by Gage.

Denote by $\Psi(w, z, u, v)$ the function

$$\phi_{111}\left(\frac{w+z}{2}, u\right)\vartheta_3^2\left(\frac{v-z}{2}\right)\vartheta_3^2\left(\frac{w-u}{2}\right).$$

Then

$$\Psi(w, z, u, v) + \Psi(w, -z, -u, -v) - \Psi(w, -z, v, u) - \Psi(w, z, -v, -u) \equiv 0$$

is an identity in w, z, u, v. The required expansions are

$$\vartheta_3(x) = \sum q^{\nu^2} \cos 2\nu x, \text{ and}$$

$$\phi_{111}(x, y) = \operatorname{ctn} x + \operatorname{ctn} y + 4 \sum q^{2n}\left[\sum \sin 2(dx + \delta y)\right].$$

For the notation in the above and in what follows, refer to I.

2. *Equivalent of Ψ-Identity.* To apply the formulas in I, §7, to the reduction of the ctn terms, make the substitution $(w, z) \rightarrow (x+y, x-y)$, and in the result apply

$$(x, y) \rightarrow ((w+z)/2, (w-z)/2).$$

Proceeding as in II, we find the following. The partitions are

$$n = 2d\delta + \nu_1^2 + \nu_2^2 + \nu_3^2 + \nu_4^2 = a_1^2 + a_2^2 + a_3^2 + a_4^2.$$

Write

$$\lambda_1 \equiv \nu_1 + \nu_2, \lambda_2 \equiv \nu_3 + \nu_4, \alpha_1 \equiv a_1 + a_2, \alpha_2 \equiv a_3 + a_4;$$

$$\sigma_1 \equiv \alpha_1 + \alpha_2, \sigma_2 \equiv \alpha_1 - \alpha_2, \sigma_{1,r} \equiv \sigma_{2,r} + 2\sigma_2,$$

$$\sigma_{2,r} \equiv 2r - 1 + e(n) - \sigma_2; S \equiv \left[\tfrac{1}{2}(|\sigma_1| - 1)\right], A \equiv \left[\tfrac{1}{2}(|\alpha_2| - 1)\right].$$

Then the identity gives