FIVE THESES ON CALCULUS OF VARIATIONS

Contributions to the Calculus of Variations, 1930. Theses submitted to the Department of Mathematics of the University of Chicago, University of Chicago Press, July, 1931.

This volume of doctoral dissertations is of interest in the first place for its mathematical content, and secondly because it indicates a possible way of meeting the publication difficulties for theses. The University of Chicago now permits "candidates to deposit either lithoprinted or printed copies of their theses in the University library." "Owing to the moderate cost of lithoprinting extra copies it seemed an opportune time to try the experiment of distributing in collected form a limited edition of those which have been submitted during the same year or a period of years, and which are concerned with the same domain of mathematics." And so we have before us a volume of 350 pages of lithographed material, containing five theses dealing with distinct topics in the calculus of variations.

The five parts of this book treat the following topics:

1. *An envelope theorem and necessary conditions for a problem of Mayer with variable end points*, by M. G. Boyce, (pages 1–44).

2. *An historical and critical study of the fundamental lemma of the calculus of variations*, by Aline Huke, (pages 45–160).

3. *A new theory of parametric problems in the calculus of variations*, by F. L. Wren, (pages 161–194).

4. *Semi-continuity in the calculus of variations and absolute minima for isoperimetric problems*, by E. J. Mc Shane, (pages 195–244).

5. *The development of sufficient conditions in the calculus of variations*, by W. L. Duren, Jr., (pages 245–350).

In Part 1, the geometric formulation of Jacobi's necessary condition is extended to the general Mayer problem in n-space with one variable end point. The first five sections develop the multiplier-rule, and the analogues of the conditions of Weierstrass and Legendre. The general procedure is that introduced and developed by Bliss and his students; the elegance and fruitfulness of his methods become once more apparent in this work. Sections 6, 7, and 8, which contain the author's contributions to the theory, bring respectively an extension to the problem under consideration of Kneser's theorem on the envelope of a family of extremals, a geometric form of Jacobi's condition based on this theorem, and a discussion of the possibility of determining a one-parameter family of extremals which possesses an envelope with the properties needed for the application of the envelope theorem. But the difficulty which has limited the usefulness of the geometric theory in other problems again arises here; whereas it is possible to state in terms of the data of the problem conditions which insure the existence of a family of extremals with an envelope, there remains the possibility that this envelope has a singularity which interferes