KRYLOFF ON MATHEMATICAL PHYSICS

Les Méthodes de Solution Approchée des Problèmes de la Physique Mathématique. (Mémorial des Sciences Mathématiques, Fascicule XLIX.) By Nicolas Kryloff. Paris, Gauthier-Villars, 1931. 69 pp.

During the present century two important methods have been developed for dealing with the boundary value problems of mathematical physics: the method of integral equations and the approximation method of W. Ritz. Integral equations are valuable for showing the existence of a solution and determining its analytical properties, but they are not well adapted to the solution of concrete problems where numerical results are wanted. The Ritz method furnishes a direct means of finding a numerical solution that is known or assumed to exist, and is especially adapted to the solution of concrete problems.

In applying the Ritz method to a physical problem, the problem is first set up as a definite integral such that the desired solution makes this integral a minimum (or maximum), just as in the calculus of variations. In the case of a function of a single variable, $y(x)$, the integrand will usually contain y and y'. The Ritz method replaces these by y_m and y_m', where y_m is a linear combination of simple functions, of the form

$$y_m = \sum_{i=1}^{m} a_i\phi_i(x), \quad (m = 1, 2, \cdots, n, \cdots, \infty).$$

Here the $\phi_i(x)$ are easily calculated functions which satisfy the boundary conditions of the problem, as $\sin m\pi x$, $x^n(1-x^2)$, etc. The coefficients a_i are determined from the condition that the integral is to be a minimum. This condition thus gives m equations which must be solved simultaneously for the a's.

Due to the labor of solving a large number of simultaneous equations, it is practically necessary to use only a few terms (not more than 6) of the series assumed for y_m. This circumstance makes it desirable that some means exist for estimating the error committed in stopping at a given number of terms. Although Ritz applied his method to a variety of problems and even solved some that had previously defied solution by all other methods, he left no means of estimating the accuracy of his results.

Since the untimely death of its brilliant author in 1909, the Ritz method has been greatly extended in usefulness and applicability by the researches of Dr. Nicolas Kryloff, and more recently with the collaboration of his pupil and assistant, Dr. N. Bogoliouboff. Kryloff has worked out explicit expressions for the upper limit of the error in the approximate solution (by the Ritz method) of several types of differential equations occurring in mathematical physics, both ordinary and partial.

The book under review gives in rather condensed form the results of Kryloff's researches on the approximate solution of ordinary differential and difference equations of the types most frequently arising in mechanics and physics. The object of the book is "to contribute to the development . . . of methods that make it possible to judge what approximation is to be taken in