ON ANGLES IN CERTAIN METRIC SPACES*

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1. *Introduction*. In a series of articles on metrical geometry† Menger has made a study of the geometry of certain abstract metric spaces and in particular he has obtained conditions for the congruence of metric spaces with sub-sets of euclidean spaces. In a recent article‡ he suggests a system of axioms for "angle spaces" and related problems.

It will be shown in this note that a theory of angles analogous to that of euclidean space is possible for convex complete metric spaces any four points of which are congruent with four points in some euclidean space. From this certain theorems regarding tangents to simple arcs are deduced.

2. *Notation and Definitions*. A euclidean space of n dimensions will be denoted by E_n.

If a and b are two points, the symbol ab will denote the distance between them or, at times, the straight line segment joining them.

If, corresponding to a set A, there is a set A', in some E_n congruent to A, we say that A *can be imbedded* in E_n. The word congruence has its usual meaning: A is congruent to A' if there is a one to one correspondence $x \sim x'$ between the points of A and A' such that, if $x \sim x'$ and $y \sim y'$, then $xy = x'y'$. The congruence of A to A' is denoted by $A \cong A'$.

In stating a congruence between two finite sets it will be understood that the pairs of points correspond in the order written. Thus, in the congruence $a+b+c \cong a'+b'+c'$, we have $a \sim a'$, $b \sim b'$, and $c \sim c'$; the congruence $a+b+c \cong a'+c'+b'$ is a different congruence.

Likewise, if ab and $a'b'$ denote two simple arcs, $ab \cong a'b'$ if there is a one to one correspondence between their points such

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† *Untersuchungen über allgemeine Metrik*, Mathematische Annalen, vol. 100, pp. 75–163, and vol. 103, pp. 466–501.

‡ *Some applications of point-set methods*, Annals of Mathematics, vol. 32, pp. 739–760.