To validate the process, proceed from $t=2$ by mathematical induction. For $t=2$, we consider $f(x_1, x_2)$ as a function of x_1, keep x_2 fixed, and apply the summation formulas of §2. In each term of the result we then consider $f(x_1, x_2)$ as a function of x_2 and apply §2.

In exactly the same way multiple summations equivalent to symbolic products (as above) of any number of factors of one or more of the types giving the explicit forms of the function $f_{\xi ps}(n)$, $(\xi=\beta, \gamma, \eta, \rho)$, in §2 can be written out as functions of the upper limits of the summations.

By the method of proof in §§1, 2, it follows that these formulas remain true under linear transformations of the arguments of the entire functions. The like does not hold for non-linear transformations, as the product of two or more umbrae is undefined.

CALIFORNIA INSTITUTE OF TECHNOLOGY

A LOGICAL EXPANSION IN MATHEMATICS*

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1. *Introduction.* Suppose we have a finite set of objects, (for instance, books on a table), each of which either has or has not a certain given property A (say of being red). Let n, or $n(1)$, be the total number of objects, $n(A)$ the number with the property A, and $n(\overline{A})$ the number without the property A (with the property not-A or $\overline{A}$). Then obviously

$$n(\overline{A}) = n - n(A). \tag{1}$$

Similarly, if $n(A\ B)$ denote the number with both properties A and B, and $n(\overline{A}\ \overline{B})$ the number with neither property, that is, with both properties not-A and not-B, then

$$n(\overline{A}\overline{B}) = n - n(A) - n(B) + n(AB), \tag{2}$$

which is easily seen to be true.

The extension of these formulas to the general case where any number of properties are considered is quite simple, and is well

* Presented to the Society, October 25, 1930.

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