QUADRATIC PARTITIONS: PAPER III

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1. *Simple Summation Formulas.* This note is independent of partitions. It gives the general summation formulas of which a very special case might have been used to pass directly from §3 to §4 of the preceding note,* and which will be used in future. The final formulas considerably extend and generalize many in the literature of the Bernoullian and allied functions.

Let $f(x)$ be an entire function of x. Write $M\equiv[(n-1)/2]$, $N\equiv[n/2]$;

$$f_\beta(n) \equiv \sum_{r=1}^{M} f(2r-1+e(n)), \quad f_\gamma(n) \equiv \sum_{r=1}^{M}(-1)^r f(2r-1+e(n)),$$

$$f_\eta(n) \equiv \sum_{r=1}^{N}(-1)^r f(2r-e(n)), \; f_\rho(n) \equiv \sum_{r=1}^{N} f(2r-e(n)).$$

One of the pairs (β,η), (ρ,γ) is sufficient, since

$$f_\rho(2n-1) = f_\beta(2n) = \sum_{r=1}^{n-1} f(2r),$$

$$f_\rho(2n-2) = f_\beta(2n-1) = \sum_{r=1}^{n-1} f(2r-1),$$

$$f_\gamma(2n-1) = f_\eta(2n-2) = \sum_{r=1}^{n-1}(-1)^r f(2r-1),$$

$$f_\gamma(2n) = f_\eta(2n-1) = \sum_{r=1}^{n-1}(-1)^r f(2r).$$

The like applies to the more general sums $f_{\xi p s}(n)$ in §2.

It is required to express $f_\xi(n)\,(\xi=\beta,\gamma,\eta,\rho)$ as explicit functions of n. Write

$$C_\beta(n) \equiv 2e(n)f(0) + 2f(n) + 2e(n)\{f(2B') - f(-2B')\}$$
$$\cdot 2\{1-e(n)\}\{f(R') - f(-R')\},$$

* This Bulletin, this issue (vol. 38, No. 8), pp. 551–554. The notation and definitions are given in I, ibid., vol. 37 (1931), pp. 870–875.