

ON THE TRIGONOMETRIC DEVELOPMENTS OF CERTAIN DOUBLY PERIODIC FUNCTIONS OF THE SECOND KIND*

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1. *Introduction.* The class of meromorphic functions which satisfy periodicity relations of the form

$$(1) \quad f(z + 2\omega_1) = c_1 f(z), \quad f(z + 2\omega_2) = c_2 f(z),$$

where the multipliers c_1 and c_2 are independent of z , and ω_1/ω_2 is a complex number with non-vanishing imaginary part, has been named by Hermite† *doubly periodic of the second kind*. It is possible to make the study of these functions depend on others of the same type, but such that one of the multipliers, say c_1 , is unity. In what follows we shall assume, further, that the periods $(2\omega_1, 2\omega_2)$ are $(\pi, \pi\tau)$, where $\tau = a + ib$, $b > 0$.

Particular interest is attached to the functions (2) below, which belong to the category just defined. In terms of the Jacobi theta functions they have the form

$$(2) \quad \phi_{\alpha\beta\gamma}(x, y) = \vartheta_1' \frac{\vartheta_\alpha(x + y)}{\vartheta_\beta(x)\vartheta_\gamma(y)},$$

where x, y are independent complex variables, and α, β, γ are certain triads, sixteen in number, which can be selected from the numbers 0, 1, 2, 3. These functions were first discovered by Jacobi‡ and have been studied by Kronecker§, Hermite||, Teixeira¶ and others. More recently, E. T. Bell** has pointed out their importance in connection with certain results in number

* Presented to the Society, April 8, 1932.

† Hermite, *Comptes Rendus*, vol. 85 (1877), . . . vol. 94 (1882); *Annales de l'École Normale Supérieure*, (3), vol. 2, 1885, p. 303. *Oeuvres*, vol. IV, p. 190–200.

‡ Jacobi, *Werke*, vol. 2, pp. 291–351.

§ Kronecker, *Monatsberichte der Berliner Akademie*, 1881, pp. 1165–1172; *Werke*, vol. IV, pp. 309, 318.

|| Loc. cit.; see also Lerch, *Acta Mathematica*, vol. 12 (1889), pp. 51–55.

¶ Teixeira, *Journal für Mathematik*, vol. 125 (1901), pp. 301–318.

** E. T. Bell, *Transactions of this Society*, vol. 22 (1921); *Colloquium Series of this Society*, vol. 7, p. 88. *Giornale di Matematiche*, vol. 59 (1921).