and we have

$$\sum G\left(x+z,\frac{y-x}{2}-z,x\right)=\epsilon_1(n)\sum_{r=1}^{T}G(t,0,2r-e(t)),$$

where $\epsilon_1(n)=1$ or 0 according as n is or is not a square >0, and $T=[t/2]$.

Similarly, from §4, if $G_1(w, u, v)$ is $G(w, u, v)$ with the restriction of entirety in (u, v) we get

$$4\sum G_1\left(x+z,\frac{y-x}{2}-z,x\right)$$
$$=\epsilon_1(n)\left[G_1(t,0,\rho'(t))-G_1(t,0,\rho'(-t))\right].$$

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THE TRANSFORMATION OF LINES OF SPACE BY MEANS OF TWO QUADRATIC REGULI*

BY A. R. WILLIAMS

If we take two quadratic reguli, a line l meets two generators of each. To l we make correspond the other transversal of the four generators. This involutory transformation of the lines of space is one of three, quite similar in principle.† This case admits a very simple and effective algebraic treatment without the use of hyperspace.

We may take for the equations of two non-singular quadrics with real rulings $x_1^2+x_2^2-x_3^2-x_4^2=0$ and $a^2x_1^2+b^2x_2^2-c^2x_3^2-d^2x_4^2=0$. On the former lies the regulus R_1 defined by $x_1-x_3=m(x_4-x_2)$, $x_1+x_3=1/(m(x_4+x_2))$. The Plücker coordinates of a line of this regulus are

$$(1)\quad p_{12}:p_{13}:p_{14}:p_{23}:p_{42}:p_{34}$$
$$=(m^2+1):2m:(m^2-1):(m^2-1):2m:-(m^2+1).$$

The other regulus R_1' on the same quadric is given by the

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† Discussed by Mr. J. M. Clarkson in the present issue of this Bulletin, vol. 38, pp. 533–540.