NOTE ON A THEOREM DUE TO BROMWICH

BY H. L. GARABEDIAN

The following well known theorem is due to Bromwich.*

THEOREM. *Suppose* (i) *that the series* $\sum a_n$ *is summable by Cesàro means of order k to the sum s,* (ii) *that* v_n *is a function of x with the properties*

$$\left.\begin{aligned} &(\alpha) && \sum n^k \left| \Delta^{k+1} v_n \right| < K\dagger \\ &(\beta) && \lim_{n\to\infty} n^k v_n = 0 \end{aligned}\right\} \quad if \quad x > 0,$$

$$(\gamma) \qquad \lim_{x\to 0} v_n = 1,$$

where K is independent of x and n. Then the series $\sum a_n v_n$ *converges if x is positive, and*

$$\lim_{x\to 0} \sum a_n v_n = s.$$

I propose to establish this theorem by a more direct and shorter method than that used by Bromwich. Moreover, this proof affords a method of exhibiting a k-fold summability with infinite matrix of reference, analogous to well known definitions of summability with finite matrices of reference which make use of repeated means, for any v_n which satisfies the conditions of the theorem under discussion.

By hypothesis the series $\sum a_n$ is summable by Cesàro means of order k, so that if

$$S_n^{(k)} = \binom{n+k-1}{k-1} s_0 + \binom{n+k-2}{k-1} s_1 + \cdots + \binom{k-1}{k-1} s_n$$

and

$$A_n^{(k)} = \binom{n+k}{k},$$

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* Mathematische Annalen, vol. 65 (1907–08), pp. 350–369; p. 359.

† Since all of the terms in the series $\sum n^k \left|\Delta^{k+1} v_n\right|$ are positive, this condition implies the convergence of the series.