SOME INVOLUTORIAL LINE TRANSFORMATIONS INTERPRETED AS POINTS OF V_2 OF S_5*

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1. *Introduction.* Consider the identity

$$\text{I} \qquad V_2(x) \equiv x_1x_4 + x_2x_5 + x_3x_6 \equiv 0$$

existing among the Plücker coordinates $x_1, x_2, \cdots, x_6$ of a line in S_3 as the equation of a quadratic hypersurface in S_5. The existence of a (1,1) correspondence between the lines of S_3 and the points of V_2 is well known, as is also the representation of ruled surfaces in S_3 by curves of the same orders and genera on V_2.†

To a bundle of lines in S_3 corresponds a plane of points on V_2, and to a plane field of lines corresponds a plane of points on V_2, called ω-planes and ρ-planes, respectively, throughout this paper. The two systems of planes are each ∞^3, and there are certain relations among them. Two planes of either system have always one and only one point in common, corresponding to the line common to the two representative bundles or plane fields; and a plane of the ω-system meets a plane of the ρ-system either in no point or in a line of points, corresponding to the flat pencil common to a bundle and a plane field of S_3 when the vertex of the bundle lies on the plane of the field.

Line transformations of S_3 are, therefore, point transformations on V_2. When the point transformations on V_2 are non-linear, their fundamental elements may be of dimension 0, 1, 2, 3, and their images, or the principal elements in the transformations, may be of dimension 1, 2, 3, 4. The transformations considered below are birational, but since the equation of V_2 does not enter into the discussion of their birationality, we conclude that they are Cremona transformations for all of S_5.

2. *Three Involutorial Transformations on* V_2.

CASE 1. J. DeVries‡ discusses synthetically several involutions

* Presented to the Society, March 26, 1932.

† See, for example, W. L. Edge, *Ruled Surfaces.*

‡ *Proceedings, Koninklijke Akademie van Wetenschappen te Amsterdam*, vol. 22 (1920), pp. 478–481, 634–640.