

A PLANE, ARCWISE CONNECTED AND CONNECTED  
IM KLEINEN POINT SET WHICH IS NOT  
STRONGLY CONNECTED IM KLEINEN\*

BY R. L. WILDER

In a recent paper, † G. T. Whyburn gave an example, in three-dimensional space, of an arcwise connected and connected im kleinen point set which is not arcwise connected im kleinen, and raised the question as to whether such a set can exist in the plane; also, whether there exists in the plane a strongly connected and connected im kleinen set which is not strongly connected im kleinen. The purpose of this note is to answer both of these questions affirmatively. Indeed, there exists in the plane a point set  $M$  which is arcwise connected and connected im kleinen, but which fails *at every point* to be strongly connected im kleinen.

Let  $S$  denote the set of all points of the plane that lie within or on the square whose vertices are the points  $(0,0)$ ,  $(1,0)$ ,  $(1,1)$ ,  $(0,1)$ ; denote these vertices by  $A$ ,  $B$ ,  $C$ ,  $D$ , respectively. The open interval  $\langle AB \rangle$  is the sum of  $c$  (where  $c$  is the cardinal number of the continuum) mutually exclusive sets each of which is dense in  $AB$ . ‡ Denote the class of these sets by  $X$ .

Let  $K$  denote the class of all continua contained in  $S$ , which contain at least one point of each of the straight line intervals  $AD$  and  $BC$ , but no point of  $AB$  or  $CD$ . This class has the cardinal number  $c$ .

There exists a one-to-one correspondence,  $T$ , between the elements of the classes  $X$  and  $K$ . Let  $E(X)$  and  $E(K)$  be elements of  $X$  and  $K$ , respectively, which correspond to one another under  $T$ . Then, if  $x$  is a point of  $E(X)$ , denote by  $l_x$  the set of points of  $S$  that lie on the vertical line through  $x$ , and let  $P_x$  denote the

\* Presented to the Society, December 31, 1930.

† *Concerning points of continuous curves defined by certain im kleinen properties*, *Mathematische Annalen*, vol. 102 (1929), pp. 313–336.

‡ See Knaster and Kuratowski, *Sur les ensembles connexes*, *Fundamenta Mathematicae*, vol. 2 (1921), pp. 206–255; see especially p. 252. This paper will be referred to hereafter as S.E.C.