SUMMATION OF FOURIER SERIES*

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1. *Introduction.* My topic, the summation of Fourier series, has been dealt with in addresses delivered to this Society by C. N. Moore, G. H. Hardy, and again by C. N. Moore, the last one less than two years old.† If I venture to speak on the same subject again, it is because the field is in a state of brisk and steady development so that it is possible for me to deal with matters which have not been exhausted by previous speakers. In particular, I am happy to be able to include in my report a number of results, published and unpublished, found by J. D. Tamarkin and myself during the last few years.

The term Fourier series is used in at least five different senses in the current literature. In the present report the term signifies a trigonometrical Fourier-Lebesgue series, that is, a series of the form

$$\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx), \tag{1}$$

or, in the complex form preferred nowadays,

$$\sum_{n=-\infty}^{+\infty} f_n e^{nix} \tag{2}$$

where the coefficients are determined by

$$\left.\begin{matrix} a_n \\ b_n \end{matrix}\right\} = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(t)\left\{\begin{matrix} \cos \\ \sin \end{matrix}\right\} nt\, dt, \quad f_n = \frac{1}{2\pi}\int_{-\pi}^{+\pi} f(t)e^{-nit}dt \tag{3}$$

respectively. Here $f(x)$ is supposed to be integrable in the sense of Lebesgue in the interval $(-\pi, +\pi)$, and is extended by the convention $f(x+2\pi) = f(x)$ outside this interval.

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† The addresses of C. N. Moore have appeared in this Bulletin, vol. 25 (1918–19), pp. 258–276, and vol. 37 (1931), pp. 240–250.